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SINGULAR INTEGRODIFFERENTIAL EQUATION OF THE PROBLEM OF OUTFLOW--ETC(U)
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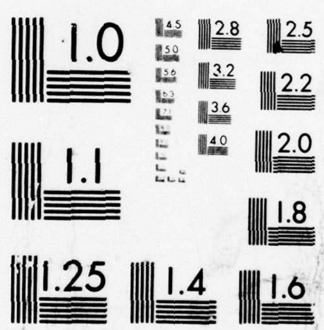


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# FOREIGN TECHNOLOGY DIVISION

SINGULAR INTEGRODIFFERENTIAL EQUATION OF THE PROBLEM OF OUTFLOW OF LIQUID FROM UNDER A SHIELD

By

E. Duysheyev





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# EDITED TRANSLATION

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| Block | Italic | Transliteration | Block | Italic | Transliteration |
|---|---|---|---|---|---|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English | Russian | English | Russian | English |
|---|---|---|---|---|---|
| sin | sin | sh | sinh | arc sh | $\sinh^{-1}$ |
| cos | cos | ch | cosh | arc ch | $\cosh^{-1}$ |
| tg | tan | th | tanh | arc th | $\tanh^{-1}$ |
| ctg | cot | cth | coth | arc cth | $\coth^{-1}$ |
| sec | sec | sch | sech | arc sch | $\text{sech}^{-1}$ |
| cosec | csc | csch | csch | arc csch | $\text{csch}^{-1}$ |

| Russian | English |
|---|---|
| rot | curl |
| lg | log |

1119

# SINGULAR INTEGRODIFFERENTIAL EQUATION OF THE PROBLEM OF OUTFLOW OF LIQUID FROM UNDER A SHIELD

E. Duysheyev.

Various problems of the outflow of liquid from under a shield, taking into account force of gravity and force of surface tension, were studied by many authors. A detailed review of works, dedicated to such problems, is available in [1, 2]. Let us examine the steady motion of incompressible liquid (taking into account the simultaneous action of forces of gravity and surface tension) between horizontal walls FA and FOE. In this case the upper wall FA on the right terminates with shield AB, slanted at angle $\alpha$ $(0<\alpha<\pi)$, and the lower - coincides with the axis of abcissa. The axis of ordinates passes through point 0 and the bottom edge B of shield AB. Liquid, with density $\rho$, flows out from under the shield with free surface BE.

Bet the magnitude of speed $v_0$ and the thickness of stream $h_0$ at point E be assigned. It is required to find speed $v$ on BE. In such a setting this problem is reduced in [3] to a system of two equations, the study of which was produced by the method of successive approximations for certain values of parameters. However, the question about the solvability of the problem for other small values of parameters in [3] remained open.

In this work the system of equations of the problem is studied for small Froude number $\varepsilon = \frac{v_0^2}{gh_0}$ characterizing the forms of the flow lines.

Let us introduce

$$\varepsilon_0 = \frac{gh_0}{v_0^2}; \quad \lambda = \frac{\sigma}{2} \frac{\nu}{h_0 v_0^2 \rho}; \quad V = \frac{v}{v_0}, \quad \frac{y}{h_0} = y, \tag{1}$$

$$u_1(t) = \varepsilon_0 (y(t) - 1), \quad u_2(t) = \lambda \theta(t). \tag{2}$$

Then from system (1.14), (1.6) of work [3] we easily obtain the system of nonlinear singular integrodifferential equations with small parameter $\varepsilon < 1$ :

$$\varepsilon \frac{du_1}{dt} = \mathcal{P}(t, u_1, \gamma, \Omega), \quad u_2(t) = \lambda [\Omega(t) - f_0(t)], \quad |t| \leq 1, \tag{3}$$

$$u_s(-1) = 0, \quad S = 1, 2. \tag{4}$$

Here for brevity of writing the following designations are used:

$$\mathcal{P}(\cdot) = -\frac{\sin[\Omega(t) - f_0(t)]}{\pi(1+t)\cdot V(u_1,\gamma)}; \quad \Omega(t) = \frac{\sqrt{1-t^2}}{\pi}\int_{-1}^{1}\frac{\ln V(u_1(x),\gamma)}{(x-t)\sqrt{1-x^2}}dx,$$

$$V(\cdot) = \sqrt{1-2u_1(t)+\gamma^2(t)} - \gamma(t), \qquad (5)$$

$$\gamma(t) = (1+t)u_1'(t), \quad u(\cdot) > 0;$$

$$f_0(t) = \frac{2\alpha}{\pi}\cdot \operatorname{arctg}\sqrt{\frac{(\alpha-1)(1+t)}{(\alpha+1)(1-t)}}, \quad \alpha > 1.$$

The solution of problem (3), (4) we will seek in the form:

$$u_1(t,\varepsilon) = W_1(t) + \varepsilon\,\eta_1(t,\varepsilon) + \Pi_1(t,\varepsilon),$$
$$\Omega(\cdot) = \omega(t) + \varepsilon T(t,\varepsilon) + q(t,\varepsilon), \qquad (6)$$

where $T(t,\varepsilon)$, $q(t,\varepsilon)$ are expressed through sought functions $\eta_1(t,s)$ and $\Pi_1(t,s)$, and $W_1(t)$ - solution of the confluent system:

$$\omega(t) = \frac{\sqrt{1-t^2}}{\pi}\int_{-1}^{1}\frac{\ln V(W_1(x),W_1')}{(x-t)\sqrt{1-x^2}}dx = f_0(t), \quad W_1(t) = 0. \qquad (7)$$

The physical sense of equations (7) involves the fact that with $\varepsilon = 0$ the free surface BE of the stream flowing out from under the shield AB is horizontal. Taking into account (4), from (7) we find:

$$W_1(t) = \frac{1}{2}\left[1 - \exp\left(-\frac{2}{\pi}\int_{-1}^{1}\left(\frac{1}{x-t} - \frac{1}{x+1}\right)f_0(x)dx\right)\right], \quad W_1(1) = 0. \quad (8)$$

Substituting now (6) in (3), we will have:

$$\varepsilon^2\eta_1'(t,\varepsilon) + \varepsilon W_1'(t) = \mathcal{P}(t, W_1 + \varepsilon\eta_1, \varepsilon\alpha_2, \omega + \varepsilon T) - \mathcal{P}(t, W_1, 0, \omega),$$
$$\varepsilon\eta_2(t,\varepsilon) = \varepsilon T(t,\varepsilon) = \frac{\lambda\sqrt{1-t^2}}{\pi}\int_{-1}^{1}\frac{\ln V(W_1 + \varepsilon\eta_1, \varepsilon\alpha_2) - \ln V(W_1(x), 0)}{(x-t)\sqrt{1-x^2}}dx, \qquad (9)$$

$$\varepsilon \Pi_1'(t,\varepsilon) = \mathcal{P}(t, \theta_1+\Pi_1, \theta_2+\tau_1, \theta_3+q) - \mathcal{P}(t, \theta_1, \theta_2, \theta_3),$$
$$\Pi_2(t,\varepsilon) = \lambda q(t,\varepsilon) \equiv \frac{\lambda\sqrt{1-t^2}}{\pi}\int_{-1}^{1} \frac{\ln V(\theta_1+\Pi_1, \theta_2+\tau_1) - \ln V(\theta_1,\theta_2)}{(x-t)\sqrt{1-x^2}}\,dx, \qquad (10)$$

$$\theta_1 = W_1(t) + \varepsilon\eta_1(t,\varepsilon), \quad \theta_2 = \varepsilon\alpha_2(t,\varepsilon) \equiv \varepsilon(1+t)\eta_2'(t,\varepsilon),$$
$$\theta_3 = \omega(t) + \varepsilon T(t,\varepsilon), \quad \tau_1(t,\varepsilon) = (1+t)\Pi_2'(t,\varepsilon). \qquad (11)$$

Let functions $\eta_s(\cdot), T(\cdot), \Pi_s(\cdot), q(\cdot)$ be represented with the aid of

$$\eta_s(t,\varepsilon) = \sum_{n=1}^{\infty} \eta_{sn}(t,\varepsilon)\varepsilon^{n-1}, \quad T(t,\varepsilon) = \sum_{n=1}^{\infty} T_n(t)\varepsilon^{n-1}; \qquad (12)$$

$$\Pi_s(t,\varepsilon) = \sum_{n=0}^{\infty} \Pi_{sn}(t,\varepsilon)\lambda_1^n, \quad q(t,\varepsilon) = \sum_{n=0}^{\infty} q_n(t,\varepsilon)\lambda_1^n, \qquad (13)$$
$$\lambda_1 = \frac{\lambda}{\pi},$$

where $\eta_{sn}(t)$ and $\Pi_{sn}(t,\varepsilon)$ satisfy Holder boundary condition, and

$$\eta_{1n}(-1) = a_n\lambda_1^{n-1}; \quad \Pi_{1n}(-1,\varepsilon) = -a_{n+1}\varepsilon^{n+1}; \quad a_\nu = \mathrm{const.} \qquad (14)$$

The problem now involves finding functions $\eta_{sn}(t)$ and $\Pi_{sn}(t,\varepsilon)$.

Let us assume that analytical, with respect to $u_1, \gamma$ and $\Omega$, functions $\mathcal{P}(\cdot)$, $\ln V(\cdot)$ together with their derivatives of $(i+k+j)$ and $(i+k)$ orders

$$P_{ikj}(t) = \frac{\mathcal{P}_{ikj}(t; W_1, 0, \omega)}{i!\,k!\,j!}, \quad \beta_{ik}(t) = \frac{B_{ik}(W_1, 0)}{i!\,k!},$$
$$B = \ln V,$$

satisfy Holder boundary condition, where from structure $\mathcal{P}(\cdot)$ and (7)

follows $P_{\tau 00}=P_{0\tau 0}=P_{\tau\kappa 0}=0$ , $\kappa,\tau \geq 1$ . Using the analyticity of functions $\mathcal{P}(\cdot), \ln V(\cdot)$ and placing (12) in (9), and then equating the coefficients with identical powers of $\varepsilon$ , we obtain a system of linear singular integral equations:

$$T_n(t) = \Psi_{0n}(t) \equiv (1+t)\left[f_{0n}(t) - \eta'_{1,n-1}(t)\right]V(W_1,0),$$
$$n \geq 1, \qquad (15)$$
$$\eta_{1n}(t) = \lambda T_n(t) \equiv \frac{\lambda\sqrt{1-t^2}}{\pi}\int_{-1}^{1}\frac{\beta_{10}(x)\eta_{1n}(x)+\beta_{01}(x)\alpha_{1n}(x)+f_{1n}(x)}{(x-t)\sqrt{1-x^2}}\,dx.$$

Here for convenience of reasonings there are used designations:

$$\alpha_{1n}(t) = (1+t)\eta'_{1n}(t), \quad \eta'_{10}(t) = W'_1(t), \quad f_{01} = f_{11} = 0,$$
$$f_{0n}(t) = \sum_{i=1}^{n-1} C_{n-1}^{i} P_{00,i+1}(t)\prod_{j=1}^{i+1} T_{\kappa_j}(t) + \sum_{2\leq i+\kappa\leq n}{}' \Big\{ P_{i0\kappa}(t) \times$$
$$\times\Big[\Big(\sum_{\alpha=1}^{n-i}\eta_{1\alpha}(t)\Big)^{i}\Big(\sum_{\alpha=1}^{n-\kappa}T_{\alpha}(t)\Big)^{\kappa}\Big] + P_{0i\kappa}(t)\Big[\Big(\sum_{\tau=1}^{n-i}\alpha_{1\tau}(t)\Big)^{i} \times$$
$$\times\Big(\sum_{\alpha=1}^{n-\kappa}T_{\alpha}(t)\Big)^{\kappa}\Big]\Big\} + \sum_{3\leq i+\kappa+j\leq n}{}' P_{i\kappa j}(t)\Big[\Big(\sum_{\alpha=1}^{n-i-1}\eta_{1\alpha}(t)\Big)^{i}\Big(\sum_{\tau=1}^{n-\kappa-1}\alpha_{1\tau}(t)\Big)^{\kappa}\Big(\sum_{\alpha=1}^{n-j-1}T_{\alpha}(t)\Big)^{j}\Big],$$
$$f_{1n}(t) = \sum_{i=1}^{n-1} C_{n-1}^{i}\Big[\beta_{i+1,0}(t)\prod_{j=1}^{i+1}\eta_{1\kappa_j}(t) + \beta_{0,i+1}(t)\prod_{j=1}^{i+1}\alpha_{1\kappa_j}(t)\Big] +$$
$$+\sum_{2\leq i+\kappa\leq n}{}' \beta_{i\kappa}(t)\Big[\Big(\sum_{\alpha=1}^{n-i}\eta_{1\kappa_j}(t)\Big)\Big(\sum_{\tau=1}^{n-\kappa}\alpha_{1\tau}(t)\Big)^{\kappa}\Big], \quad \sum_{j=1}^{i+1}\kappa_j = n,$$
$$i,\kappa,j \geq 1,$$

where sums $\sum'$ are expanded by rules indicated in [4]. From the structure of functions $T_\tau(t), \mathcal{P}(\cdot), \ln V(\cdot)$ it follows that $f_{0n}(t)$ and $f_{1n}(t)$ are limited and satisfy Holder boundary condition. Since $\Psi_{01}(t)$ - known function, then from system (15) it is possible to easily find $\eta_{11}(t)$. Let $\eta_{1\tau}(t)$, $\tau = \overline{1, n-1}$ be found. Then, by applying the theories of linear singular integral equations, for any $n \geq 1$ from expression (15) we find

$$\eta_{2n}(t)=\lambda\Psi_{0n}(t),\ \eta_{1n}(t)=-\beta_{10}^{-1}(t)\Big[\beta_{10}(-1)\eta_{1n}(-1)+ \\ +f_{1n}(-1)+\beta_{01}(t)(1+t)\lambda\cdot\Psi_{0n}'(t)+f_{1n}(t)+ \\ +\frac{1}{\pi}\int_{-1}^{1}\Big(\frac{1}{x-t}-\frac{1}{x+1}\Big)\Psi_{0n}(x)dx\Big]. \tag{16}$$

Similarly, from the analyticity of functions $\mathcal{P}$, $\ln V$ and (10), (13) we will have the following recurrence equations for

$\Pi_{1n}(t,\varepsilon)$ :

$$\varepsilon\Pi_0'(t,\varepsilon)=\sum_{i+k\geq 1}A_{i0k}(t,\varepsilon)\Pi_{10}^i(t,\varepsilon)q_0^k(t,\varepsilon),\ \Pi_{10}(t,5)=0,$$

$$q_0(t,\varepsilon)=\frac{\sqrt{1-t^2}}{\pi}\int_{-1}^{1}\frac{\sum_{i=1}^{\infty}b_{i0}(x,\varepsilon)\Pi_{10}^i(x,\varepsilon)}{(x-t)\sqrt{1-x^2}}dx, \tag{17}$$

$$\varepsilon\Pi_{1n}'(t,\varepsilon)-a_{100}(t,\varepsilon)\Pi_{1n}(t,\varepsilon)-\frac{a_{001}(t,\varepsilon)\sqrt{1-t^2}}{\pi}\cdot \\ \times\int_{-1}^{1}\frac{b_{10}(x,\varepsilon)\Pi_{1n}(x,\varepsilon)}{(x-t)\sqrt{1-x^2}}dx=F_n(t,\varepsilon), \tag{18}$$

$$\Pi_{2n}(t,\varepsilon)=\pi q_{n-1}(t,\varepsilon),\qquad n\geq 1,$$

$$q_n(\cdot)=\frac{\sqrt{1-t^2}}{\pi}\int_{-1}^{1}\frac{b_{10}(x,\varepsilon)\Pi_{1n}(x,\varepsilon)}{(x-t)\sqrt{1-x^2}}dx+\Psi_n(t,\varepsilon),$$

$$\Psi_n(\cdot)=\frac{\sqrt{1-t^2}}{\pi}\int_{-1}^{1}\frac{b_{01}(x,\varepsilon)\tau_{2n}(x,\varepsilon)+Q_n(x,\varepsilon)}{(x-t)\sqrt{1-x^2}}dx,$$

$$Q_n(\cdot)=\sum_{i=1}^{n-1} C_{n-1}^{i}\left[B_{i+1,0}(t,\varepsilon)\prod_{j=1}^{i+1}\Pi_{1k_j}(t,\varepsilon)+B_{0,i+1}(t,\varepsilon)\times\right.$$

$$\left.\times\prod_{j=1}^{i+1}\tau_{2k_j}(t,\varepsilon)\right]+\sum_{2\leq i+k\leq n}{}' B_{ik}(t,\varepsilon)\left[\left(\sum_{\alpha=1}^{n-k}\Pi_{1\alpha}(t,\varepsilon)\right)^{i}\left(\sum_{\alpha=1}^{n-i}\tau_{2\alpha}(t,\varepsilon)\right)^{k}\right],$$

$$\sum_{j=1}^{i+1}k_j=n,$$

$$F_n(\cdot)=a_{010}(t,\varepsilon)\tau_{2n}(t,\varepsilon)+a_{001}(t,\varepsilon)\cdot\Psi_n(t,\varepsilon)+$$

$$+\sum_{i=1}^{n-1}C_{n-1}^{i}\left[a_{i+1,0,0}(t,\varepsilon)\prod_{j=1}^{i+1}\Pi_{1k_j}(t,\varepsilon)+a_{0,i+1,0}(t,\varepsilon)\prod_{j=1}^{i+1}\tau_{2k_j}(t,\varepsilon)\right]+$$

$$+\sum_{2\leq i+k\leq n}{}'\left\{a_{ik0}(t,\varepsilon)\left[\left(\sum_{\alpha=1}^{n-k}\Pi_{1\alpha}\right)^{i}\cdot\left(\sum_{\alpha=1}^{n-i}\tau_{2\alpha}\right)^{k}\right]+a_{0ik}(t,\varepsilon)\times\right.$$

$$\left.\times\left[\left(\sum_{\alpha=1}^{n-k}\tau_{2\alpha}\right)^{i}\cdot\left(\sum_{\alpha=1}^{n-i}q_{\alpha}\right)^{k}\right]\right\}+\sum_{3\leq i+k+j\leq n}{}' a_{ikj}(t,\varepsilon)\left[\left(\sum_{\alpha=1}^{n-k-j}\Pi_{1\alpha}\right)^{i}\times\right.$$

$$\left.\times\left(\sum_{\alpha=1}^{n-i-j}\tau_{2\alpha}\right)^{k}\cdot\left(\sum_{\alpha=1}^{n-i-k}q_{\alpha}\right)^{j}\right],$$

$$a_{\tau\nu\varkappa}(t,\varepsilon)=\sum_{i+k+j\geq 2}A_{ikj}(t,\varepsilon)C_i^{\tau}\Pi_{10}^{i-\tau}C_j^{\varkappa}q_0^{j-\varkappa};\quad i\geq\tau,\ j\geq\varkappa,$$

$$B_{\tau k}(t,\varepsilon)=\sum_{i+k\geq 2}d_{ik}(t,\varepsilon)C_i^{\tau}\Pi_{10}^{i-\tau}(t,\varepsilon),$$

$$\tau_{2\alpha}=(1+t)\cdot\Pi_{1\alpha}'(t,\varepsilon),$$

where $A_{ikj}(\cdot)$, $d_{ik}(\cdot)$ - derivatives of $(i+k+j)$ and $(i+k)$ orders of function $\mathcal{P}$ and $\ln V$ with respect to dependent variable.

Since, by virtue of the structure of formulas (5), functions $A_{ikj}(\cdot)$ are integrated and $d_{ik}(\cdot)$ are continuous according to Holder, and the right side of equation (17) satisfies Lifshits condition, then equation (17) is solvable.

Let us assume that $A_{100}(t,\varepsilon)<0$, $a_{100}(t,\varepsilon)<0$ and the solution of equation (17) is found [4].

For assigned n = 1 from (18) it is not difficult to determine $\Pi_{s1}(t,\varepsilon)$. Let functions $\Pi_{si}(t,\varepsilon)$, $i=\overline{1,n-1}$ already be found. Then $\Pi_{sn}(\cdot)$, and consequently $F_n(t,\varepsilon)$, - is known function. Considering that the first equation of system (18) can easily lead to an equation of Fredholm type, the solution of which is presented in the form:

$$\Pi_{1n}(t,\varepsilon)=Q_{1n}(t,\varepsilon)+\int_{-1}^{1}R(t,x,\varepsilon)\,Q_{1n}(x,\varepsilon)\,dx,\ n\geq 1, \quad (19)$$

where $R(\cdot)$ - resolvent of kernel

$$H(\cdot)=\frac{f(t,\varepsilon)\,\beta_{10}(x,\varepsilon)}{\pi\varepsilon\sqrt{1-x^2}}\int_{-1}^{1}f^{-1}(\tau,\varepsilon)\,\frac{\sqrt{1-\tau^2}\cdot a_{001}(\tau,\varepsilon)}{x-\tau}\,d\tau,$$

$$f(t,\varepsilon)=\exp\left(\frac{1}{\varepsilon}\int_{-1}^{t}a_{100}(s,\varepsilon)\,ds\right),\ a_{100}(\cdot)<0,$$

$$Q_{1n}(\cdot)=f(t,\varepsilon)\left[\Pi_{1n}(-1,\varepsilon)+\varepsilon^{-1}\int_{-1}^{t}f^{-1}(x,\varepsilon)\,F_n(x,\varepsilon)\,dx\right].$$

Estimates for functions $\eta_{sn}(t)$, $\Pi_{sn}(t,\varepsilon)$ and convergence of series (12) and (13) are established similarly [4].

Thus, by knowing functions (8), (12), (13), (16), (13) and (19), we can determine the solution of initial problem by formula (6), where with $\varepsilon\to 0$ $U_s(t,\varepsilon)$ strives toward the solution of confluent system. On the basis of (6), (8), (12), (13), (15) and (18), (19) the magnitude of velocity $V(\cdot)=V_0(t,\alpha,\varepsilon,\lambda)$, and consequently, $\Omega(t)=\Omega_0(t,\alpha,\varepsilon,\lambda)$

is known function of t, depending also on $\alpha$ - angle of slope of shield AB, small Froude number $\varepsilon < 1$ and parameter $\lambda$, considering the coefficient of surface tension. Omitting detail, let us present the formula of the coefficient of compression of the stream in the following form:

$$K(\alpha,\varepsilon,\lambda)=\frac{\pi}{\pi+2}\,\frac{1}{1-J(\alpha,\varepsilon,\lambda)},\quad J(\cdot)=\frac{2}{\pi-2}\left[1-R(\alpha,\varepsilon,\lambda)\right]^{*}, \quad (20)$$

$$R(\cdot)=-\frac{1}{2}\int_{-1}^{1}\frac{\sin\left[f_0(t)-\Omega_0(t,\alpha,\varepsilon,\lambda)\right]}{(1+t)V_0(t,\alpha,\varepsilon,\lambda)}\,dt,\quad J(\cdot)<1.$$

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